Solutions - Homework 1

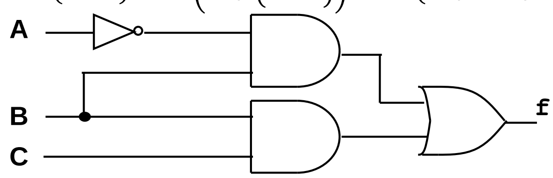
(Due date: January 21st @ 5:30 pm)
Presentation and clarity are very important!

PROBLEM 1 (25 PTS)

a) Simplify the following functions using ONLY Boolean Algebra Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (12 pts)

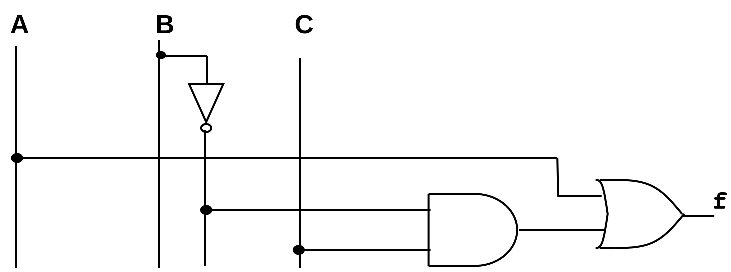
- ✓ $F = \overline{A(\overline{B}\oplus\overline{C}) + \overline{B}}$
- ✓ $F(X,Y,Z) = \prod(M_1, M_3, M_6, M_7)$
- ✓ $F = (\overline{C}+\overline{B})(C+A)(\overline{B}+A) + CA$
- ✓ $F = \overline{(\overline{X+Z})Y + X\overline{Y}Z}$

✓ $F = \overline{A(\overline{B}\oplus\overline{C}) + \overline{B}} = \overline{A(\overline{B}\oplus\overline{C})}.B = \left(\overline{A} + \left(\overline{\overline{B}\oplus\overline{C}}\right)\right).B = (\overline{A} + BC + \overline{B}\,\overline{C}).B = \overline{A}B + BC + B\overline{B}\,\overline{C} = \overline{A}B + BC$

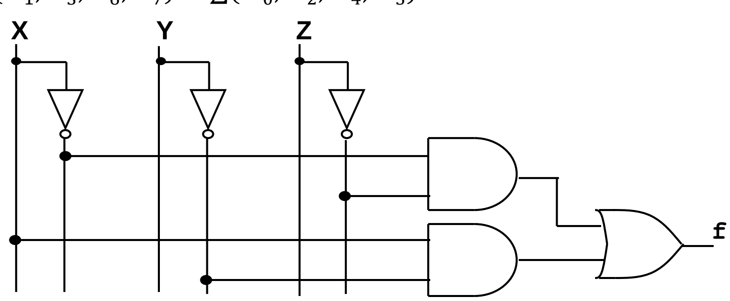


✓ $F = (C+A)(\overline{C}+\overline{B})(A+\overline{B}) + CA = (C+A)(\overline{C}+\overline{B}) + CA$

$(C+A)(\overline{C}+\overline{B}) + CA = C\overline{B} + \overline{C}A + A\overline{B} + CA = C\overline{B} + \overline{C}A + CA = C\overline{B} + A$



✓ $F(X,Y,Z) = \prod(M_1,M_3,M_6,M_7) = \sum(m_0,m_2,m_4,m_5) = \overline{X}\,\overline{Y}\,\overline{Z} + \overline{X}Y\overline{Z} + X\overline{Y}\,\overline{Z} + X\overline{Y}Z = \overline{X}\,\overline{Z} + X\overline{Y}$

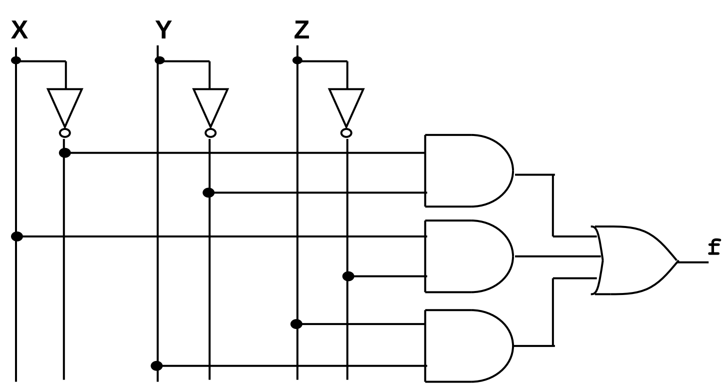


✓ $F = \overline{(\overline{X+Z})Y + X\overline{Y}Z} = \overline{(\overline{X+Z})Y}.\overline{X\overline{Y}Z} = (X+\overline{Y}+Z)(\overline{X}+Y+\overline{Z})$

$F = X\overline{X} + XY + X\overline{Z} + Z\overline{X} + ZY + Z\overline{Z} + \overline{Y}\,\overline{X} + \overline{Y}Y + \overline{Y}\,\overline{Z} = XY + X\overline{Z} + Z\overline{X} + ZY + \overline{Y}\,\overline{X} + \overline{Y}\,\overline{Z}$

$F = X\overline{Z} + \overline{X}\,\overline{Y} + \overline{Y}\,\overline{Z} + XY + Z\overline{X} + ZY = X\overline{Z} + \overline{X}\,\overline{Y} + XY + Z\overline{X} + ZY$

$F = X\overline{Z} + XY + YZ + \overline{Y}\,\overline{X} + \overline{X}Z = X\overline{Z} + XY + YZ + \overline{Y}\,\overline{X} = ZY + \overline{Z}X + XY + \overline{Y}\,\overline{X} = ZY + \overline{Z}X + \overline{Y}\,\overline{X}$



b) Based on the formula $x\oplus y = x\bar{y} + \bar{x}y$, demonstrate that $(a\oplus b)\oplus c = a\oplus(b\oplus c) = b\oplus(a\oplus c)$. You can express each function using the canonical sum of products, or complete the truth table for each function. (5 pts)

$(a\oplus b)\oplus c = (a\bar{b} + \bar{a}b)\oplus c = \overline{(a\bar{b} + \bar{a}b)}c + (a\bar{b} + \bar{a}b)\bar{c} = (ab + \bar{a}\bar{b})c + (a\bar{b} + \bar{a}b)\bar{c} = abc + \bar{a}\bar{b}c + a\bar{b}\bar{c} + \bar{a}b\bar{c} = \sum m(7,1,4,2)$.
$a\oplus(b\oplus c) = a\oplus(b\bar{c} + \bar{b}c) = a(bc + \bar{b}\bar{c}) + \bar{a}(b\bar{c} + \bar{b}c) = abc + a\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}\bar{b}c = \sum m(7,4,2,1)$.
$b\oplus(a\oplus c) = (a\oplus c)\oplus b = (a\bar{c} + \bar{a}c)\oplus b = (ac + \bar{a}\bar{c})b + (a\bar{c} + \bar{a}c)\bar{b} = acb + \bar{a}\bar{c}b + a\bar{c}\bar{b} + \bar{a}c\bar{b} = \sum m(7,2,4,1)$.
* Note that $x\oplus y = y\oplus x$

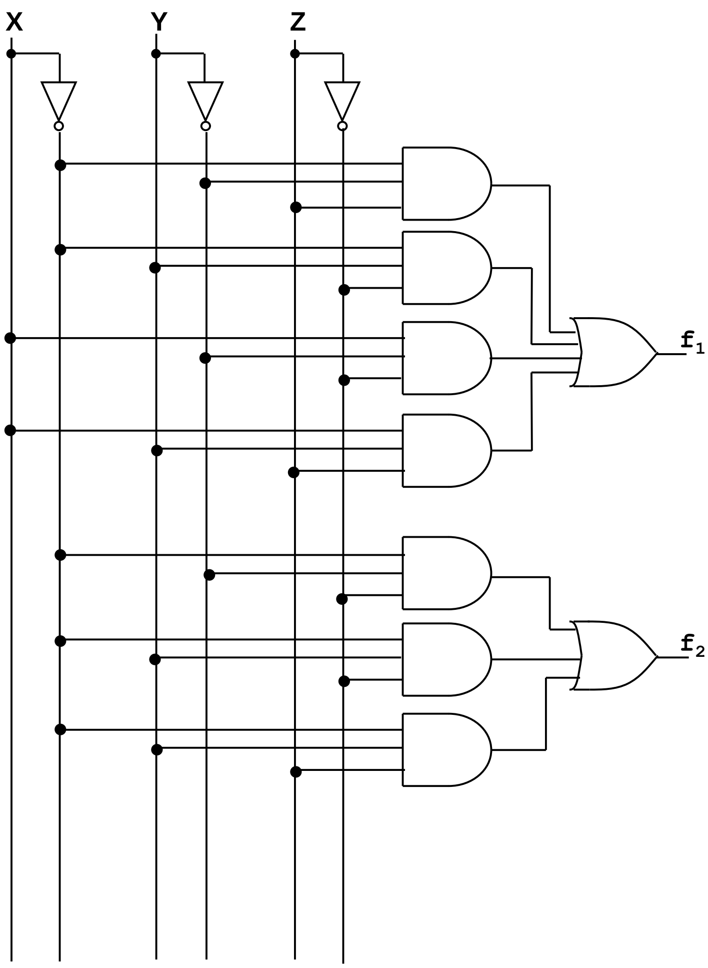
c) For the following Truth table with two outputs: (8 pts)
- Provide the Boolean functions using the Canonical Sum of Products (SOP), and Product of Sums (POS).
- Express the Boolean functions using the minterms and maxterms representations.
- Sketch the logic circuits as Canonical Sum of Products and Product of Sums.

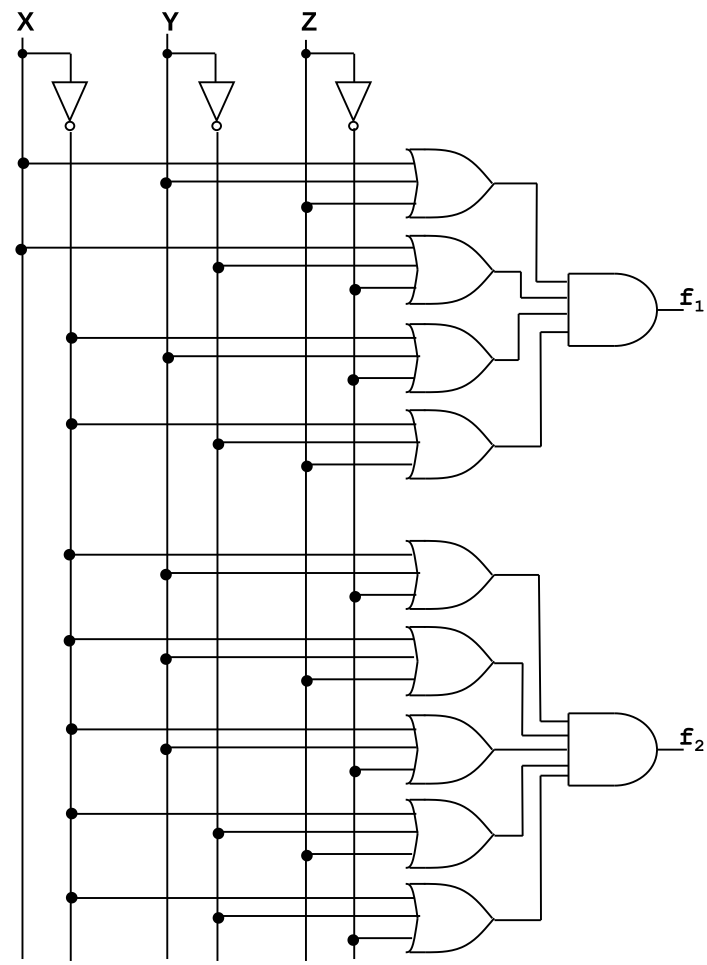
x	y	z	f_1	f_2
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	0

Sum of Products
$f_1 = \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XYZ$
$f_2 = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + \bar{X}YZ$

Product of Sums
$f_1 = (X + Y + Z)(X + \bar{Y} + \bar{Z})(\bar{X} + Y + \bar{Z})(\bar{X} + \bar{Y} + Z)$
$f_2 = (X + Y + \bar{Z})(\bar{X} + Y + Z)(\bar{X} + Y + \bar{Z})(\bar{X} + \bar{Y} + Z)(\bar{X} + \bar{Y} + \bar{Z})$

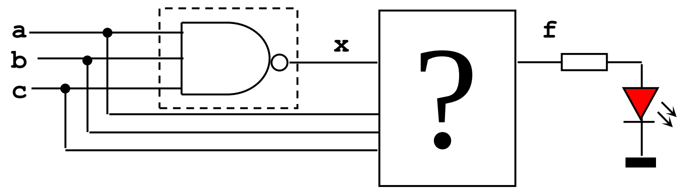
Minterms and maxterms: $f_1 = \sum(m_1, m_2, m_4, m_7) = \prod(M_0, M_3, M_5, M_6)$.
$f_2 = \sum(m_0, m_2, m_3) = \prod(M_1, M_4, M_5, M_6, M_7)$.





PROBLEM 2 (10 PTS)

- Design a circuit (simplify your circuit) that verifies the logical operation of a 3-input NAND gate. f = '1' (LED ON) if the NAND gate does NOT work properly. Assumption: when the NAND gate is not working, it generates 1's instead of 0's and vice versa.



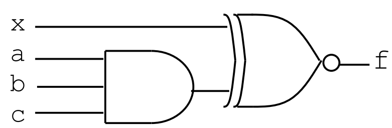
x	a	b	c	f
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

bc \ xa	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	1	0	1	0
10	1	1	0	0

$$f = \bar{b}\bar{x} + \bar{x}\bar{c} + \bar{x}\bar{a} + xabc$$

$$f = \bar{x}\left(\bar{b} + \bar{c} + \bar{a}\right) + xabc$$

$$f = \bar{x}\left(\overline{bca}\right) + xabc = \overline{x \oplus abc}$$



PROBLEM 3 (15 PTS)

- Complete the truth table for a circuit with 4 inputs x, y, z, w that activates an output ($f = 1$) when the number of 1's in the inputs is odd. For example: If $xyzw = 1100 \rightarrow f = 0$. If $xyzw = 1011 \rightarrow f = 1$.
- Provide the Boolean function using the minterm representation.
- Sketch the logic circuit using ONLY 2-input NAND gates. Tip: try to simplify the function using XOR gates.

x	y	z	w	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

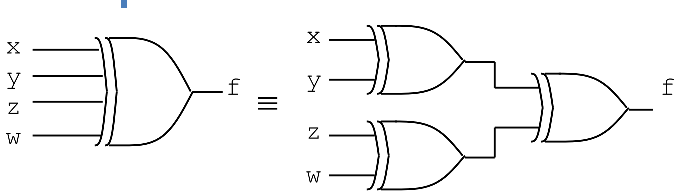
zw \ xy	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

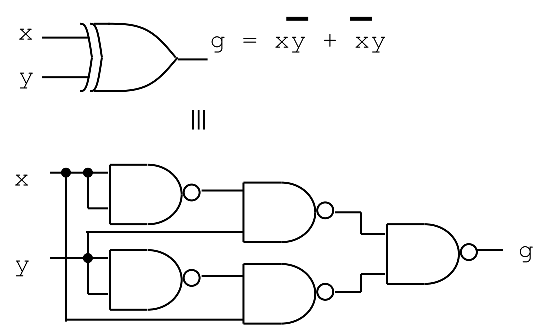
$$f = \bar{x}\bar{y}\bar{z}w + \bar{x}\bar{y}z\bar{w} + \bar{x}y\bar{z}\bar{w} + \bar{x}yzw + xy\bar{z}w + xyz\bar{w} + x\bar{y}\bar{z}\bar{w} + x\bar{y}zw$$

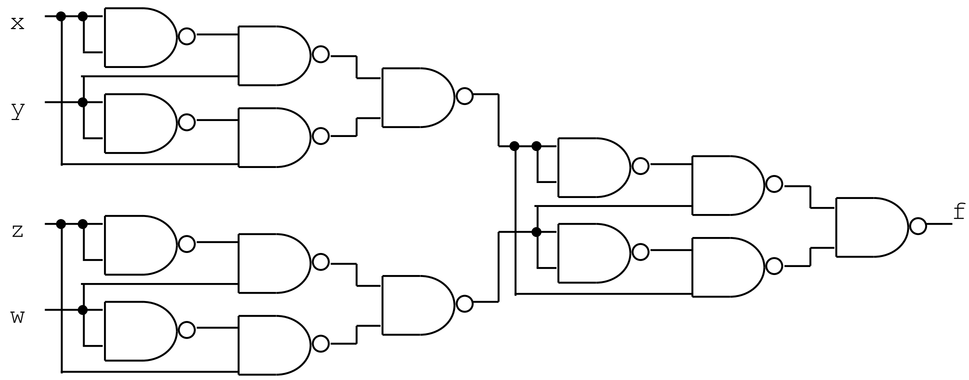
$$f = \bar{x}\bar{y}(z \oplus w) + \bar{x}y(\overline{z \oplus w}) + xy(z \oplus w) + x\bar{y}(\overline{z \oplus w})$$

$$f = (\overline{x \oplus y})(z \oplus w) + (x \oplus y)(\overline{z \oplus w})$$

$$f = (x \oplus y) \oplus (z \oplus w) = (x \oplus y \oplus z \oplus w)$$



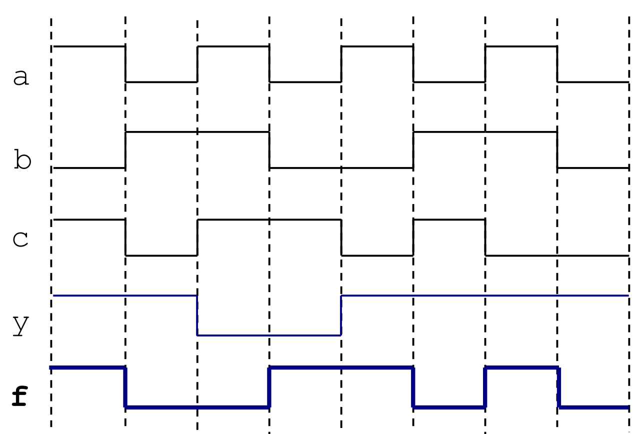




PROBLEM 4 (20 PTS)

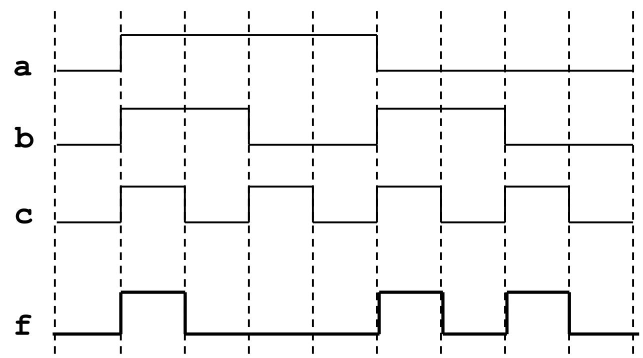
a) Complete the timing diagram of the logic circuit whose VHDL description is shown below: (5 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
         f: out std_logic);
end circ;

architecture st of circ is
  signal x, y: std_logic;

begin
  x <= not(a xor b);
  y <= x nand c;
  f <= y xor (not a);
end st;
```



b) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the logic circuit that generates this waveform. Then, complete the VHDL code. (10 pts)

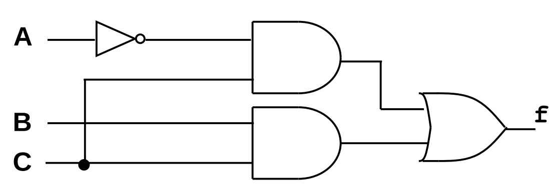
```
library ieee;
use ieee.std_logic_1164.all;

entity wav is
  port ( a, b, c: in std_logic;
               f: out std_logic);
end wav;

architecture st of wav is

begin
  f <= (not(a) and c) or (b and c);
end st;
```



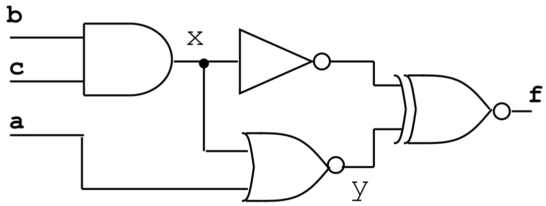
a	b	c	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

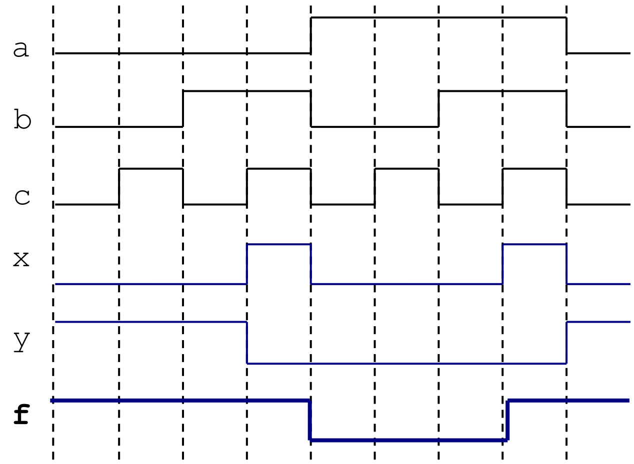
c \ ab	00	01	11	10
0	0	0	0	0
1	1	1	1	0

$$f = \overline{a}c + bc$$



c) Complete the timing diagram of the following circuit: (5 pts)

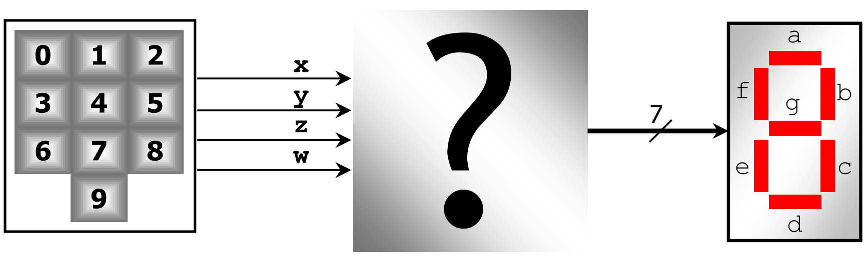




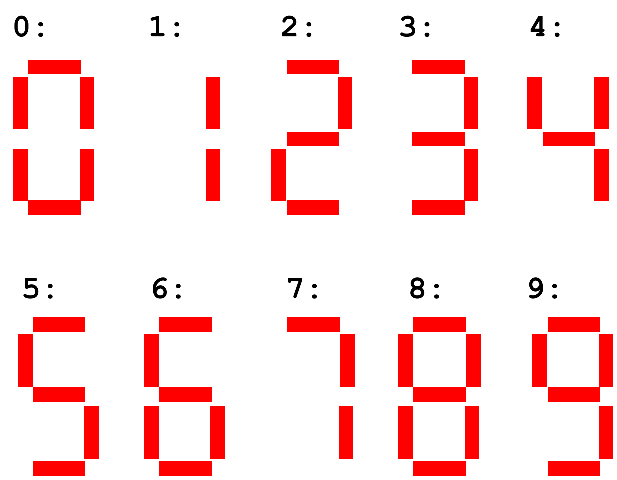
PROBLEM 5 (30 PTS)

- A numeric keypad produces a 4-bit code as shown below. We want to design a logic circuit that converts each 4-bit code to a 7-segment code, where each segment is an LED: A LED is ON if it is given a logic '1'. A LED is OFF if it is given a logic '0'.

✓ Complete the truth table for each output (a, b, c, d, e, f, g).
✓ Provide the simplified expression for each output (a, b, c, d, e, f, g). Use Karnaugh maps for a, b, c, d, e and the Quine-McCluskey algorithm for f, g. Note it is safe to assume that the codes 1010 to 1111 will not be produced by the keypad.



Value	X Y Z W	a b c d e f g
0	0 0 0 0	
1	0 0 0 1	
2	0 0 1 0	
3	0 0 1 1	
4	0 1 0 0	
5	0 1 0 1	
6	0 1 1 0	
7	0 1 1 1	
8	1 0 0 0	
9	1 0 0 1	1 1 1 1 0 1 1
	1 0 1 0	
	1 0 1 1	
	1 1 0 0	
	1 1 0 1	
	1 1 1 0	
	1 1 1 1	



Value	X	Y	Z	W	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	X	X	X	X	X	X	X
	1	0	1	1	X	X	X	X	X	X	X
	1	1	0	0	X	X	X	X	X	X	X
	1	1	0	1	X	X	X	X	X	X	X
	1	1	1	0	X	X	X	X	X	X	X
	1	1	1	1	X	X	X	X	X	X	X

a

zw \ xy	00	01	11	10
00	1	0	X	1
01	0	1	X	1
11	1	1	X	X
10	1	1	X	X

b

zw \ xy	00	01	11	10
00	1	1	X	1
01	1	0	X	1
11	1	1	X	X
10	1	0	X	X

c

zw \ xy	00	01	11	10
00	1	1	X	1
01	1	1	X	1
11	1	1	X	X
10	0	1	X	X

d

zw \ xy	00	01	11	10
00	1	0	X	1
01	0	1	X	1
11	1	0	X	X
10	1	1	X	X

e

zw \ xy	00	01	11	10
00	1	0	X	1
01	0	0	X	0
11	0	0	X	X
10	1	1	X	X

$a = \bar{y}\bar{w} + wy + x + z$
$b = \bar{y} + \bar{z}\bar{w} + zw$
$c = y + \bar{z} + w$
$d = x + z\bar{w} + \bar{y}z + \bar{w}\bar{y} + \bar{z}wy$
$e = \bar{w}\bar{y} + z\bar{w}$

- $f = \sum m(0,4,5,6,8,9) + \sum d(10,11,12,13,14,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
0	m_0 = 0000 ✓	$m_{0,4}$ = 0-00 ✓ $m_{0,8}$ = -000 ✓	**$m_{0,4,8,12}$ = --00** ~~$m_{0,8,4,12}$ = --00~~	
1	m_4 = 0100 ✓ m_8 = 1000 ✓	$m_{4,5}$ = 010- ✓ $m_{4,6}$ = 01-0 ✓ $m_{4,12}$ = -100 ✓ $m_{8,9}$ = 100- ✓ $m_{8,10}$ = 10-0 ✓ $m_{8,12}$ = 1-00 ✓	**$m_{4,5,12,13}$ = -10-** ~~$m_{4,12,5,13}$ = -10-~~ **$m_{4,6,12,14}$ = -1-0** ~~$m_{4,12,6,14}$ = -1-0~~ **$m_{8,9,10,11}$ = 10--** ~~$m_{8,10,9,11}$ = 10--~~ **$m_{8,9,12,13}$ = 1-0-** ✓ ~~$m_{8,12,9,13}$ = 1-0-~~ **$m_{8,10,12,14}$ = 1--0** ✓ ~~$m_{8,12,10,14}$ = 1--0~~	**$m_{8,10,12,14,9,13,11,15}$ = 1---** ~~$m_{8,9,12,13,10,14,11,15}$ = 1---~~
2	m_5 = 0101 ✓ m_6 = 0110 ✓ m_9 = 1001 ✓ m_{10} = 1010 ✓ m_{12} = 1100 ✓	$m_{5,13}$ = -101 ✓ $m_{6,14}$ = -110 ✓ $m_{9,11}$ = 10-1 ✓ $m_{9,13}$ = 1-01 ✓ $m_{10,11}$ = 101- ✓ $m_{10,14}$ = 1-10 ✓ $m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{9,13,11,15}$ = 1--1** ✓ **$m_{10,14,11,15}$ = 1-1-** ✓	
3	m_{11} = 1011 ✓ m_{13} = 1101 ✓ m_{14} = 1110 ✓	$m_{11,15}$ = 1-11 ✓		
4	m_{15} = 1111 ✓			

$f = \bar{z}\bar{w} + y\bar{z} + y\bar{w} + x\bar{y} + x$

Prime Implicants		Minterms					
		0	4	5	6	8	9
$m_{0,4,8,12}$	$\bar{z}\bar{w}$	**X**	X			X	
$m_{4,5,12,13}$	$y\bar{z}$		X	**X**			
$m_{4,6,12,14}$	$y\bar{w}$		X		**X**		
$m_{8,9,10,11}$	$x\bar{y}$					X	**X**
$m_{8,10,12,14,9,13,11,15}$	x					X	**X**

$f = \bar{z}\bar{w} + y\bar{z} + y\bar{w} + x$

- $g = \sum m(2,3,4,5,6,8,9) + \sum d(10,11,12,13,14,15)$.
  Too many minterms. We better optimize: $\bar{g} = \sum m(0,1,7) + \sum d(10,11,12,13,14,15)$

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
0	m_0 = 0000 ✓	**$m_{0,1}$ = 000-**		
1	**m_1 = 0001**			
2	m_{10}= 1010 ✓ m_{12}= 1100 ✓	$m_{10,11}$ = 101- ✓ $m_{10,14}$ = 1-10 ✓ $m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{10,14,11,15}$ = 1-1-** ~~$m_{10,11,14,15}$ = 1-1-~~ ✓ **$m_{12,14,13,15}$ = 11--** ~~$m_{12,13,14,15}$ = 11--~~ ✓	
3	m_7 = 0111 ✓ m_{11}= 1011 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	**$m_{7,15}$ = -111** $m_{11,15}$ = 1-11 ✓ $m_{13,15}$ = 11-1 ✓ $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

$\bar{g} = \bar{x}\bar{y}\bar{z}w + \bar{x}\bar{y}\bar{z} + yzw + xz + xy$

Prime Implicants		Minterms		
		0	1	7
m_1	$\bar{x}\bar{y}\bar{z}w$		X	
$m_{0,1}$	$\bar{x}\bar{y}\bar{z}$	**X**	X	
$m_{7,15}$	yzw			**X**
$m_{10,14,11,15}$	xz			
$m_{12,14,13,15}$	xy			

$\bar{g} = \bar{x}\bar{y}\bar{z} + yzw \quad \Rightarrow \quad g = (x + y + z)(\bar{y} + \bar{z} + \bar{w})$